Highlights

Competition level as a key parameter in renewable energy auctions: insights from an Agent-Based Model.

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- An agent-based model for renewable energy auctions is proposed.
- A phase transition on the level of competition ρ is detected at the critical value $\rho = 2$.
- The proposed model allows to recover with good accuracy the result of the German wind on-shore and solar PV auctions, and of the Argentine solar PV auction.
- Methods to control prices based on adjustment of the competition level and the ceiling prices are proposed and evaluated with the aid of the model.

Competition level as a key parameter in renewable energy auctions: insights from an Agent-Based Model.

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ABSTRACT

We propose a simple approach to auctions based on statistical mechanics tools and evolutionary game theory to assess the impact of the competition level on the prices. A sealed-bid, pay-as-bid scheme with several rounds is considered. At each round bidders place their bids following a normal distribution with fixed variance and mean value μ characteristic of each bidder. Bidders learn from round to round, adjusting myopically their μ according to their performance in the round. We study the resulting dynamics using agent-based simulations, and we identify a phase transition depending on the competition level of the auction. Our model is in contrast with the classical literature on auctions which assumes bidders act purely rationally. Despite the simplicity of the model, it can explain the increasing and decreasing trends of the outcomes of real auctions, like the wind onshore and solar PV energy auctions held in Germany from 2017 to 2022, and the solar PV auction held in Argentina between 2016 and 2018.

1. Introduction

Auctions are a very popular mechanism to allocate all kinds of goods (art, natural resources like renewable energies, telecommunication licenses, any kind of 'stuff' on internet,...). They are quite complex to study since they can be thought of as games with incomplete information: players (i.e., bidders) try to bid to maximize their utility in a context where they usually do not know with certainty the value the other participants (and possible themselves) attribute to the auctioned goods and so ignore their bidding behaviour. Moreover, beyond the classical ascending auction widely used in fine art auctions where the auctioneer raises the price until only one participant remains and is declared the winner, there exist multiple auction formats specifying the bidding process, who wins and how much the participants pay, presenting various pros and cons from the bidders' and the auctioneer's point of view.

There is a rich mathematical theory of auctions mainly developed by researchers coming from economic disciplines (see, e.g., Klemperer (2004) and Krishna (2002)) like the former Nobel prizes W. Vickrey (1996), R. Myerson (2007), P. Milgrom and R. Wilson (2020). Bidders are assumed to bid rationally so as to maximize their expected utility. The main questions are then the existence of an equilibrium (i.e., how rational bidders are expected to bid), the expected revenue of the auctioneer, and the allocation of the goods to the bidders who most value them or not.

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In this paper we are especially motivated by renewable energy (RE) auctions, which are procurement auctions. In classical auctions (like fine art auctions), the auctioneer sells a good and bidders state the price at which they are willing to acquire it. In a RE auction (and in general procurement auctions) the opposite happens: the auctioneer, usually a government or a public institution, offers to buy a given amount of energy and bidders offer to sell to the auctioneer a certain volume of energy at a given price. Bidders want to sell energy for a high price while the auctioneer wants to buy it for a low price.

Auctions are becoming the most widely used mechanism to allocate RE, like solar photo-voltaic, wind, geothermal, or hydro. Indeed more than a hundred countries had used auctions at least once by the end of 2018 (IRENA (2019)), and the European Union made auctions mandatory to grant support from member states governments since 2017 Szabó, Bartek-Lesi, Dézsi, Diallo, Mezösi, Kitzing, Woodman, Fitch-Roy, del Rio, Resch, von Blücher, Wigand, Menzies and Anatolitis (2020). The use of RE auctions led to an overall significant decrease of the price of energy (IRENA (2019); Szabó et al. (2020)).

Beside the abstract auction format specifying the bidding process, the winner determination and the payment rule, other design parameters can impact the success of an RE auction like, e.g., the ceiling price (a cap on the maximum bid such that any higher bid is discarded), or pre-qualifications and requisites that the bidders must fulfill. Combined to those abstract design parameters, local and global politico-socio-economic factors have a decisive impact on the results of the auction. For instance, the economic characteristics of the bidders like their cost of energy production, or the possibility of obtaining loans which is key in developing economies, are affected due to the country's risk qualifications and credit access. Due to their obvious high socio-economic impact and their complexity, RE auctions are currently the subject of an intense research, and we refer the interested reader to the survey del Río and Kiefer (2021); Kruger, Eberhard and Kyle (2018); MEDREG (2019); Szabó et al. (2020); Wigand, Förster and Silvana Tiedemann (2016) and the references therein.

RE auctions can be held in repeated rounds (like e.g., Germany, Argentina and Brazil) thus giving the possibility to the participants of learning and adapting their behaviour from round to round to improve their future pay-off. However the possibly large number of participants and their heterogeneity usually pose serious difficulty in using theoretical results from evolutionary game theory. To circumvent this difficulty, agent-based models (ABM) are increasingly used to study some aspects of auctions (Anatolitis and Welisch (2017); Azadeh, Ghaderi, Nokhandan and Sheikhalishahi (2012); Lundberg (2019); Welisch (2018)). Indeed their bottom-up approach is well suited to aggregate microscopic behaviour of heterogeneous agents up to their macroscopic consequences. In fact, their flexibility makes them a very attractive tool to study the implications of particular policy decision, see e.g. Castro, Drews, Exadaktylos, Foramitti, Klein, Konc, Savin and van den Bergh (2020); Holtz and Chappin (2019); Hansen, Liu and Morrison (2019); Holtz, Schnülle, Yadack, Friege, Jensen, Thier, Viebahn and Chappin (2020); Niamir, Filatova, Voinov and Bressers (2018) about the use of ABM to model climate-energy policies. Focusing on the use of ABM to study auctions, the authors in Anatolitis and Welisch (2017) and Lundberg (2019) are strongly motivated by the wind and solar auctions that have been held in Germany since 2017. They both propose ABM to understand different aspects of these auctions: the heterogeneity of the participants with respect to their economic size Anatolitis and Welisch (2017), and the influence of the payment rule, pay-as-bid or uniform Lundberg (2019). These models greatly differ in their complexity and their underlying assumption concerning the participants. On one hand the model Anatolitis and Welisch (2017) has many parameters to account for the heterogeneity of the participants mainly with respect to their production capacity and cost. Bidders are moreover assumed to be highly rational: they determine their behaviour optimizing their expected discounted total payoff from the current round to the last one. On the other hand, the model in Lundberg (2019) is quite simple with only two parameters, one for the learning process of the bidders and the other relative to their evaluation of the auctioned energy source. Bidders in this model learn in a myopic way as opposed to Anatolitis and Welisch (2017). Both models lead to interesting insights and policy consequences, and they are the main motivation for the present work. Notice that as in any model with bounded rationality, there is no guarantee that agents learn the Nash equilibrium, so they can reach a suboptimal equilibrium, or keep on changing strategies without converging to a stationary state.

This ABM approach is widely used in the study of complex systems in physics, and physicists have applied tools from statistical mechanics to study several social and economic models. In these models, individuals or firms are reduced to simple dummy particles, and random encounters between them are identified with shocks during which they exchange energy (in this case, any quantity of interest from an economic or social point of view). These random microscopic shocks then lead to macroscopic observable changes at the whole population level. Notice that individuals in these models are usually not rational: they do not behave to maximize an expected payoff, on the contrary they always apply the same rule to exchange some kind of energy. Although clearly simplistic, this approach has obtained great success in reproducing empirical facts with simple models amenable to analysis. For instance V. Pareto observed at the end of the 19th century Pareto (1896) that the distribution of wealth in a given society is remarkably stable through time and space. It was shown (see, e.g., Chakrabarti, Chakraborti, Chakravarty and Chatterjee (2013)) that very simple -and not rationally motivated- wealth exchange rules between members of a society can lead to wealth distributions sharing similar characteristics to those observed empirically, see also Pinasco, Cartabia and Saintier (2018) were such a result can be obtained with slightly rational individuals. Notice also that experiments show that humans do not always behave in a purely rational way (Kagel and Levin (2014)). This statistical mechanics approach to Economy and Social Sciences (Castellano, Fortunato and Loreto (2009)) gave rise to two fast growing areas, Sociophysics and Econophysics (see Sen and Chakrabarti (2014); Slanina (2013)). On the other hand, a strong mathematical formalism was built to describe these phenomena in terms of kinetic equations, see Bellomo (2008); Aylaj, Bellomo, Gibelli and Reali (2020); Pareschi and Toscani (2013) and the references therein.

It is well known that a high number of participants is an important factor to have a competitive auction and obtain low energy prices (see e.g. Klemperer (2002)). As Bulow and Klemperer wrote in Bulow and Klemperer (1994), *No amount of bargaining power is as valuable to the seller as attracting one extra bona fide bidder*. In the context of RE auctions, the work Wigand et al. (2016) examines the impact of the level of competition, among other factors, in the success and failure of several RE auctions held across the world. More theoretical considerations can be found in Kreiss (2016) where the author first highlights the importance of the composition of the bidders population to properly assess the competition level, and then study the impact of various auction parameters on the level of competition from a qualitative point of view. It would be thus desirable to know how competitive an auction must be to obtain low prices, and in general, to assess quantitatively the importance of the competition level in the outcome of the auction and test Bulow and Klemperer dictum in the context of RE auctions.

The main objective of this paper is to answer this question. Our starting point are the wind on-shore energy auctions held in Germany from 2017 to 2022. They are particularly suitable for our puprpose due to the fluctuation of the competition level, and we can observe a strong and clear correlation between the competition levels and the resulting prices. To deepen our analysis and obtain a quantitative estimation of the impact of the competition level, we then propose a model of auction taking place in several rounds. By incorporating ideas from evolutionary game theory, participants will be able to adapt (learn) from round-to-round but only in a limited myopic way, thus following the econophysics philosophy and taking into account bidders do not act in general purely rationnally. We show in particular that the level of competition in the auction is a key parameter of the model in the sense it determines both bidders' long-time behaviour and the evolution of the prices.

The paper is organized as follows. We first present in the next section the wind on-shore auctions held in Germany since 2017 and highlight the influence of the participation level as our main motivation for the present paper. We then describe our model in Section §3. The resulting dynamics is studied in Section §5 through agent-based simulations. Though very simple, we show in Section §7 that our model is able to reproduce with good accuracy the outcomes of real auctions like the German wind on-shore, German solar PV and Argentine solar PV auctions. We conclude the paper in Section §8 analyzing policy consequences of our findings.

2. Renewable energy auctions in Germany

Following the *Guidelines on state aid for environmental protection and energy 2014–2020*, by the European Commission (2014), the public support to renewable energy in Germany is determined by auctions since 2015. From 2015 to 2019, a total of 17.25 GW of renewable energy capacity has been added in 40 auction rounds mainly involving PV solar and wind off- and on-shore. A detailed description of the schemes and their outcomes can be found in Sach, Lotz and von Blücher (2019). The German experience has been the subject of numerous publications (Anatolitis and Welisch (2017); Batz Liñeiro and Müsgens (2021); Grashof, Berkhout, Cernusko and Pfennig (2020); Kácsor (2021); Lundberg (2019); Sach et al. (2019); Szabó et al. (2020); Welisch (2018); Welisch and Kreiss (2019)) where several aspects like the price obtained, the realization rate, the impact of the payment rule.... have been studied using agent-based modelling and game theory. The large numbers of rounds that have been held and the public availability of data from the webpage of the German government Federal Ministry for Economic Affairs and Climate Action certainly contributes to this popularity.

We will focus in this paper on the wind on-shore auctions. During the year 2017, the German government implemented a set of rules aiming at fostering the participation of *community energy projects* (BEG, Bürgerenergiegesellschaften) defined as wind cooperatives with at least ten private individuals. Project size of BEG were limited but benefited of substantial advantages compared to other participants such as lower material pre-qualification requirement, reduced penalty in case of non-realization, a longer realization period. Moreover BEG were awarded with the highest awarded bid instead of their own bid price. A detailed study of this special mixed auction scheme was done in Lundberg (2019).

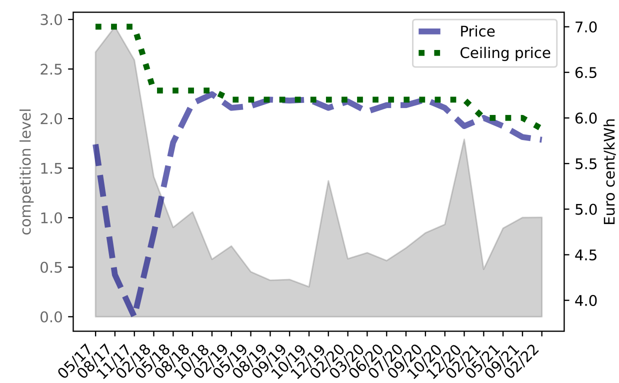

Figure 1: Evolution from round to round of the competition level ρ (grey) defined in (1), the ceiling price (dotted green line) and the mean price (dashed blue line) for the wind on-shore auction in Germany based on data available at Bundesnetzagentur.

We show in Figure 1 the evolution from round to round of the average winning price (blue slashed curve) form 2017 to 2022. We can observe the prices dropped down during 2017 and started from 2018 rising up steadily to reach the ceiling price (red dotted curve). Certainly the abandonment of the BEG special conditions played a role in the abrupt change of price trend.

To gain insights, it is useful (see Figure 3 in Szabó et al. (2020)) to plot on the same figure as a grey shadow the competition level ρ defined as the ratio of the total volume offered by bidders over the total auctioned volume, namely

$$\rho = \frac{\text{total offered volume}}{\text{total auctioned volume}}. \tag{1}$$

We can observe a strong correlation between the price evolution and the level of competition. In particular it seems that when the level of competition is below 200% (i.e., the auctioned volume is greater than half the offered volume), then the price increases, whereas it decreases when the level of competition is higher than 200%. The same effect can also be observed in the German solar PV auctions shown in Figure 2, though less clearly. Indeed until Feb. 2018, and from March 2019. until Oct. 2019, the price decreases and the competition level stays above 200%. The two changes in the trend of the prices occur in Feb. 2018 and March 2019 where the competition level pass below 2.

To assess the impact of the competition level ρ on the outcomes and the possible threshold at $\rho = 2$, we propose in the following a simple agent-based model of auctions.

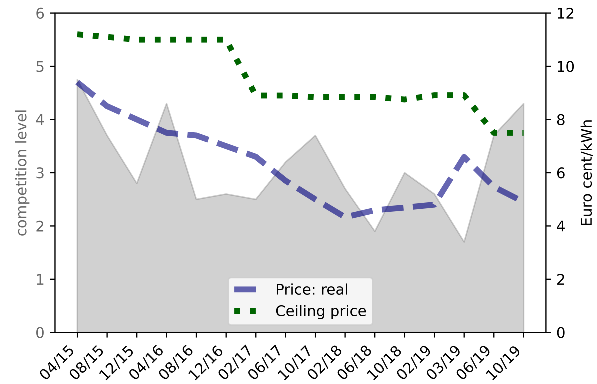

Figure 2: Evolution from round to round of the competition level ρ (grey) defined in (1), the ceiling price (dotted green line) and the mean price (dashed blue line) for the solar PV auction in Germany based on data available at Bundesnetzagentur.

3. Model description

We consider a sealed-bid auction taking place in several rounds $t = 1, \dots, T$ involving the same N bidders. Denote $c_1, ..., c_N$ their cost.

A round t of the auction is organized as follows:

Step 1 The auctioneer publicly announces the total auctioned volume V_t and the ceiling price CP_t of the round

Step 2 Bidding process: each bidder i submits a sealed bid b_i, the price at which he or she is willing to provide a volume v of energy. Bidders are all assumed to submit the same volume v, and we suppose for simplicity that V_t is a multiple of v. There are then $N_{w,t} := V_t/v$ winners at round t.
The bid b_i is drawn at random from a normal distribution $N(\mu_i, \sigma^2)$. Notice $b_i - c_i$ is then the mark-up (or profit) of bidder i, and $\mu_i - c_i$ its expected value. If bidder i's bid is greater than the ceiling price, i.e. $b_i > CP_t$, then to have an admissible bid, we put $b_i := CP_t$. If bidder i's bid is less than his/her cost, i.e. $b_i < c_i$, then bidder i abandons the round since it will not be profitable. The mean bid μ_i will change from round to round as explained in the learning step below. The variance σ^2 is assumed to be the same for all bidders through the whole auction.

Step 3 Determination of the winners: the auctioneer sorts the bids $b_1, ..., b_N$ in ascending order. The bidders who submitted the lowest $N_{w,t}$ bids are the winners.

Step 4 Learning: bidder i updates their parameter μ_i to μ_i' first calculating the expected *relative markup* as $(\mu_i - c_i)_+/(CP_t - c_i)$. It represents his/her expected profit upon winning but scaled relatively to the maximum possible profit. Here $x_+ = max\{x, 0\}$ is the positive part of a real number x. Notice the expected relative markup belongs to $[0, 1]$. Bidder i then updates it adding $+\gamma$ if he/she won or $-\gamma$ otherwise. Here $\gamma > 0$ is the learning parameter, the same for every bidder and round. Eventually bidder i computes μ_i' scaling the relative markup back to the interval $[c_i, CP_{t+1}]$ with CP_{t+1} the ceiling price of the next round:

$$\mu_i' := \begin{cases} c_i + \left(\frac{(\mu_i - c_i)_+}{CP_t - c_i} + \gamma\right)(CP_{t+1} - c_i) & \text{if bidder } i \text{ won,} \\ c_i + \left(\frac{(\mu_i - c_i)_+}{CP_t - c_i} - \gamma\right)(CP_{t+1} - c_i) & \text{if bidder } i \text{ lost.} \end{cases} \tag{2}$$

This rule models a myopic behaviour in the sense that bidders lower or increase their relative mark-up only taking into account how well they performed in the round, thus reinforcing good behaviour and penalizing bas ones. The reinforcement of actions leading to good outcomes is a robust properties observed in experimental psychology on both human and animal learning which has been succesfully used in games and economic literature - see e.g. the seminal paper Roth and Ido (1995). This idea is by now very popular in algorithmic game theory (see e.g. the multiplicative weight algorithm to obtain no-regret algorithm - chap. 17 in Roughgarden (2016)) and computation (see e.g. the clasical book S. and G. (2018)).

We summary the workflow of a round in the flow chart shown in Figure 3.

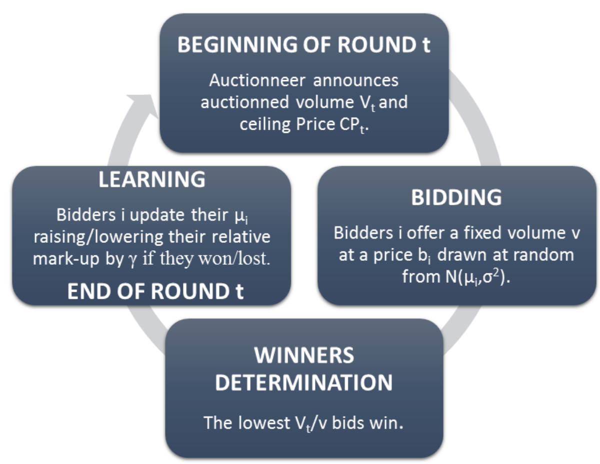

Figure 3: Flow chart of a round in our model.

Notice that no communication among bidders is allowed during the whole auction. Bids are sealed and bidders only know their own performance in the round. A bidder's learning process is thus only affected by the other bidders through their result.

In the next sections we study the dynamics via agent-based simulations focusing on the evolution from round to round of the distribution of μ among the bidders and the winning bids. In particular we will be most interested in studying the impact of the competition level ρ_t of round t defined in (1) and of the bidding noise σ on the long time evolution of the distribution of μ in the bidders population. Notice that since the total auctionned and offered volume are V_t and Nv respectiveley and the number of winners is $N_{w,t} = V_t/v$ we can rewrite ρ_t as

$$\rho_t := N/N_{w,t}, \tag{3}$$

To do so it will be convenient from a theoretical point of view to assume γ small. Indeed γ being the learning rate mainly fixes the time scale of the dynamic. Assuming it small means that bidders are conservative changing their behaviour smoothly. Observable effects will then only appear when T is large. This is of course unrealistic but will prove very useful concerning the impact of the competition level. Moreover taking γ small also presents theoretical advantages. Indeed a mean field approximation allows to write down a partial differential equation for the evolution of the distribution of the parameter μ in the bidders population. The rigorous derivation of this equation and its analysis can be found in Kind, Pinasco and Saintier (2021), since the study of this equation is non-trivial. Although the use of differential equations is quite unusual in the study of auctions (but not in evolutionary game theory), we believe that it is a valuable tool which can bring deep insights into agent-based models going far beyond simulations. Indeed the methodology we just described is widely used in the physic and applied mathematics communities to study social and economic phenomena (see e.g. Pareschi and Toscani (2013); Pérez-Llanos, Pinasco and Saintier (2021, 2020); Pérez Pérez, Pinasco, Saintier and Silva (2018); Vazquez, Saintier and Pinasco (2020); Saintier, Pablo Pinasco and Vazquez (2020)). In the present case, an analysis of this equation provides a theoretical explanation of the threshhold $\rho = 2$ we will observe numerically in the next section.

4. Model results

In this section we present agent-based simulations of the dynamics described in the previous section. In all the simulations we took $N = 1000$ bidders. Since we are mainly interested in the bidding dynamics, we suppose for simplicity that the bidders have all the same cost 0, i.e. $c_1 = ... = c_N = 0$, and that the ceiling price is $CP_t = 1$ for any round t. The updating rule (2) of bidders' μ parameter then simplifies to

$$\mu_i' := \begin{cases} ((\mu_i)_+ + \gamma) & \text{if bidder } i \text{ won,} \\ ((\mu_i)_+ - \gamma) & \text{if bidder } i \text{ lost.} \end{cases} \tag{4}$$

Bidders' μ parameters are initially drawn at random independently and uniformly in $[0, 1]$. We set the learning parameter to $\gamma = 0.001$. We run the dynamic for $T = 1200$ rounds.

We begin by taking $\sigma = 0$, so that bidders bid exactly their μ. To assess the influence of the competition level ρ, we assume it constant throughout the auction. We show in Figure 4 the evolution from round-to-round of the μ parameter of the N bidders (Left panel), and the evolution of the mean winning bid (Right panel) for two competition level values: $\rho = 4$ (Top) and $\rho = 1.25$ (Bottom).

We can observe that bidders coordinate in the sense that they tend to have all the same value of the μ parameter (up to random fluctuation of order γ), say μ_∞. This is quite surprising since, as mentioned before, no explicit communication between bidders is allowed in our model. This common value then converges either to the ceiling price 1 when $\rho = 1.25$ or to the cost 0 when $\rho = 4$. The same qualitative behaviour is observed for other initial distribution of the μ parameter among the bidders (see Appendix).

Further simulations varying the value of ρ and the initial condition show that bidders always coordinate and the limit μ_∞ of the common μ value is always, up to fluctuation of order γ, very close to 0 or 1 depending on the value of ρ. Suppose for instance that initially the bidders are divided into three groups of proportion $1/4$, $1/4$, and $1/2$, with μ parameter uniformly distributed at random in $[0, 0.25]$, $[0.4, 0.6]$, and $[0.65, 1]$, respectively. We show in Figure 5 the evolution of the μ of each bidder and of the mean winning bid for two different competition level values $\rho = 4$ and $\rho = 1.25$ and three different noise level $\sigma = 0$, $\sigma = 0.1$ and $\sigma = 0.5$. We can clearly see that bidders coordinate and that μ_∞ is close to 0 when $\rho = 1.25$, and to 1 when $\rho = 4$.

To further study the dependence of μ_∞ w.r.t ρ, we plot in Figure 6 the final common value μ_∞ as a function of ρ. The values were averaged over 10 runs. We can clearly observe a transition at $\rho = 2$: when $\rho < 2$, the final μ value is close to 1, whereas it is close to 0 for $\rho > 2$.

We now examine the influence of the noise σ. Recall that participants submit bids b drawn from a normal distribution $b \sim N(\mu, \sigma^2)$ where μ is specific to each participant and changes from round to round, and σ is the same for all participants. We can thus think of b as μ perturbed by a noise $N(0, \sigma^2)$. We show in Figure 7 the evolution from round to round of the μ parameter of the bidders and of the mean winning bid for the two competition level values $\rho = 4$ and $\rho = 1.25$, and for two different noise values: $\sigma = 0.1$ (top), $\sigma = 0.5$ (bottom). We can observe a similar qualitative behaviour as for $\sigma = 0$ in the sense that participants still coordinate their μ and the common final value μ depends on the level of competition with a sharp transition at $\rho = 2$. Figure 6 shows the final common value of μ as a function of the competition level ρ for different values of σ. It confirms that $\rho = 2$ is the threshold value whatever the noise level (notice however that noise seems to delay bidder's coordination).

Our agent model thus allows to precisely quantify the importance of the competition level ρ showing a sharp threshold at $\rho = 2$ dividing two opposite tendencies: below, prices go down to the cost, and above, prices go up to the ceiling price. The threshold value is moreover indifferent to the bidding noise σ thus suggesting it could be observed in real auctions. We examine this issue in the next section, when we compare the results of our model with the German wind on-shore and solar PV auctions, and Argentine solar PV auctions.

Let us briefly mention that our model can be studied mathematically by using a mean field approximation with

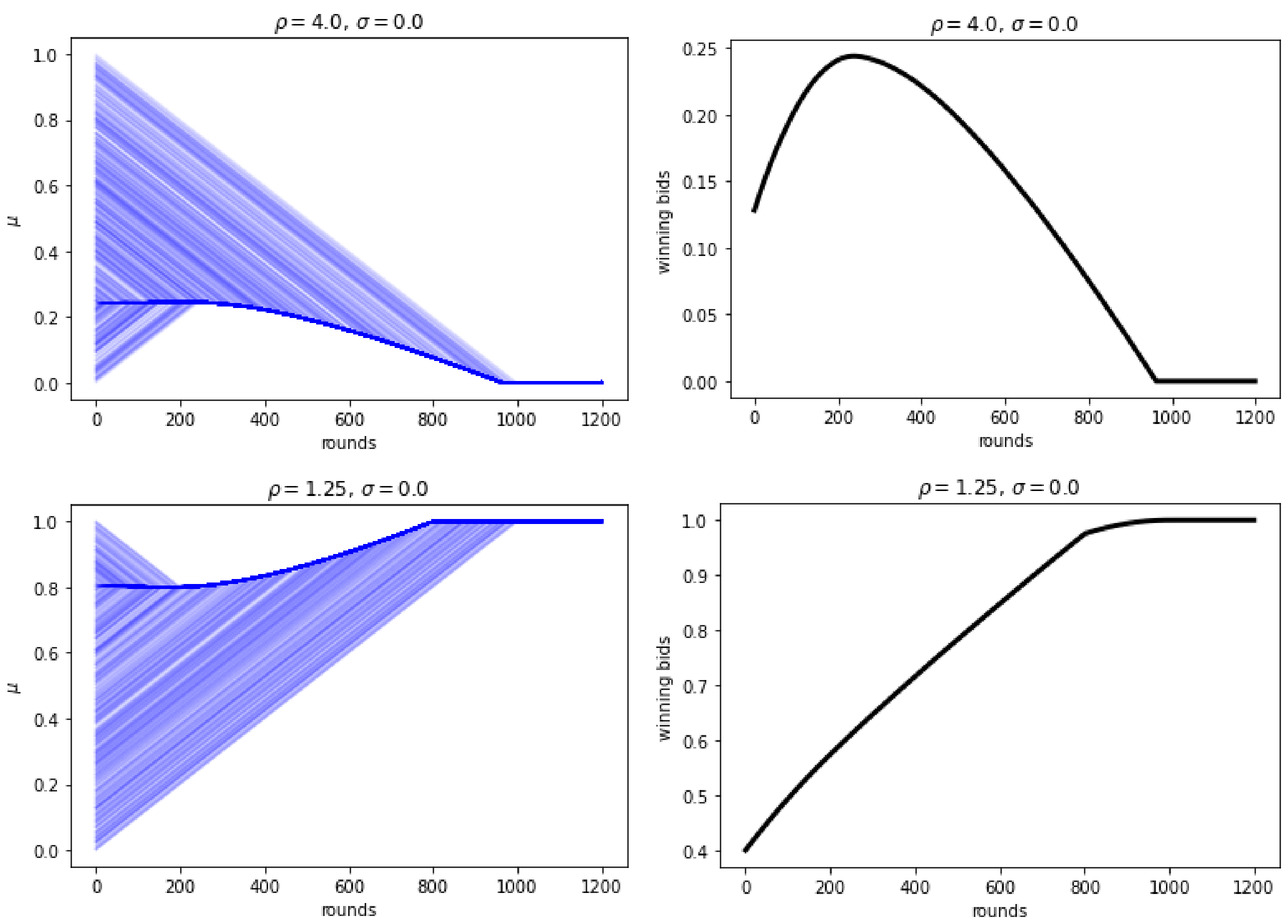

Figure 4: Evolution from round-to-round of the μ values of bidders (left), and of the mean winning bid (right) for two constant competition level values: $\rho = 4$ (top) and $\rho = 1.25$ (bottom).

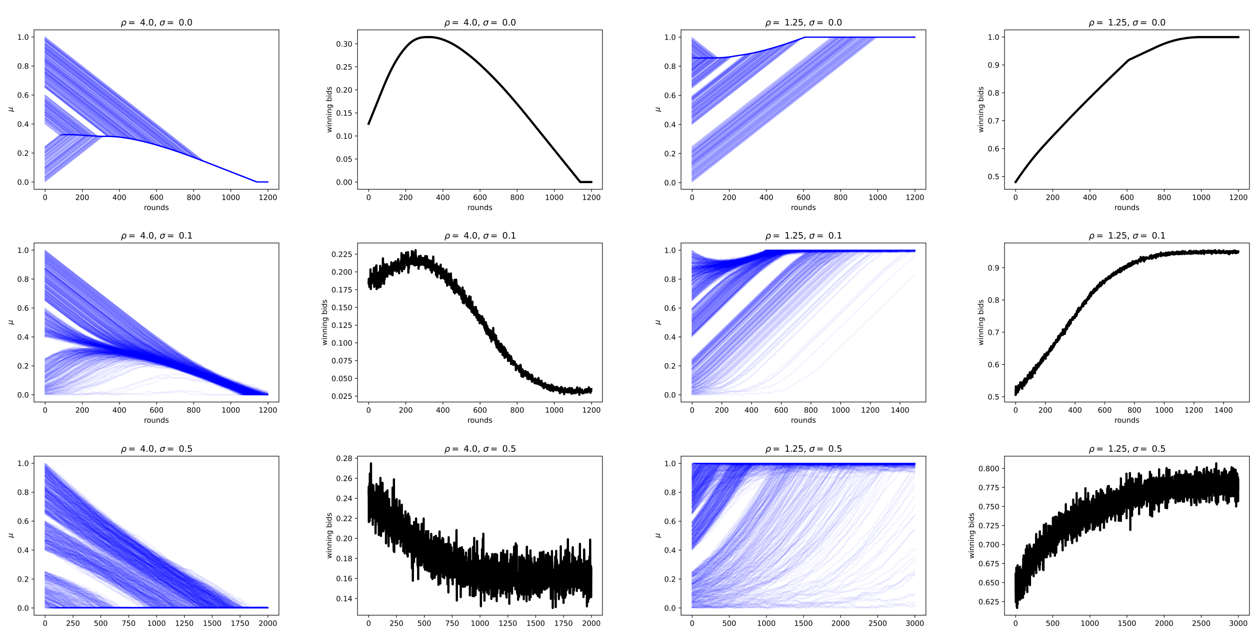

Figure 5: Evolution from round to round of the μ values of bidders (blue curves - columns 1 and 3), and of the mean winning bid (black curve - columns 2 and 4) for two competition level values $\rho = 4$ (columns 1 and 2) and $\rho = 1.25$ (columns 3 and 4), and for three different values of σ: $\sigma = 0$ (top) $\sigma = 0.1$ (middle) and $\sigma = 0.5$ (bottom). Parameter μ is initially uniformly distributed among the bidders as in the text.

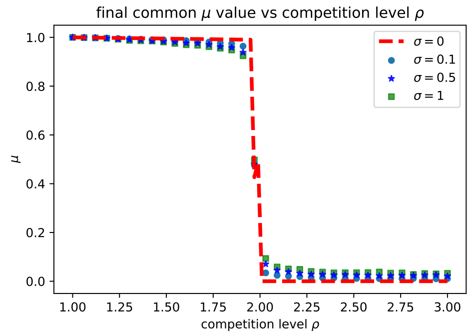

Figure 6: Final common value of μ as a function of the competition level ρ for different values of noise σ.

partial differential equations. Indeed, when $\sigma = 0$, the distribution $f_t(\mu)$ of the μ's in the bidders population at time t can be shown to solve the following equation

$$\frac{\partial f_t(\mu)}{\partial t} + \frac{\partial}{\partial \mu}\Big[(2P_0[f_t](\mu) - 1)f_t(\mu)\Big] = 0 \qquad \text{in } (0,1)$$

in the limit $N \to +\infty$ and $\gamma \to 0$. Here

$$P_0[f_t](\mu) = \begin{cases} 1_{F_t(\mu)\le p} + \dfrac{p - f_t([0,\mu))}{f_t(\{\mu\})} 1_{\mu = F_t^{-1}(p)} & \mu \in (0,1], \\ 1_{f_t(\{0\})\le p} + \dfrac{p}{f_t(\{0\})} 1_{f_t(\{0\})>p} & \mu = 0, \end{cases}$$

is the probability a bidder with μ wins, $F_t(\mu)$ is the proportion of bidders with parameter less than or equal to μ, and $p := 1/\rho$. This term translates the learning rule (4) and thus drives the dynamics in $(0,1)$. Let us note that the phase transition is clearly observed in the term $2P_0[f_t] - 1$. Informally, the population reaches a consensus about the true value μ, that drives the population to the lower bound if the probability that a bidder wins is $P_0[f_t] < 1/2$, and to the ceiling price if $P_0[f_t] > 1/2$. The rigorous derivation of this equation and its analysis can be found in Kind et al. (2021). Although the use of differential equations is quite unusual in the study of auctions (but not in evolutionary game theory), we believe that it is a valuable tool which can bring deep insights into agent-based models going far beyond simulations. Indeed the methodology we just described is widely used in the physic and applied mathematics communities to study social and economic phenomena (see e.g. Pareschi and Toscani (2013); Pérez-Llanos et al. (2021, 2020); Pérez Pérez et al. (2018); Vazquez et al. (2020); Saintier et al. (2020))

5. Comparison of our model with real auctions.

To assess the strengths and limitations of our model, we consider three real auctions, Germany wind on-shore and solar PV auctions and Argentine solar PV auction. For each of these auctions w examine to what extent our model is able to reproduce the global trends of the evolution of prices. The choice of these auctions is motivated by two reasons: the availability of data, and the number of rounds of the scheme to allow the learning process of our model to take place.

5.1. German wind on-shore and solar PV auctions.

Concerning the German wind on-shore auction, we already observed in Section §2 a strong correlation between the competition level ρ and the prices. More precisely we noticed that the value $\rho = 2$ seemed to determine the evolution of the prices, which agrees with the phae transioon our model exhibits at $\rho = 2$. We now want to see if the evolution of prices between 2017 and 2020 can reproduced by our model. With the real competition level and ceiling price of the Germany wind auction, we simulated the agent-based model with $N = 100$ bidders. We assume that their cost is uniformly distributed between 2 and 7, and a bidder with cost c has an initial mark-up chosen uniformly at random between 2 and 5. The best fit was obtained using a learning rate $\gamma = 0.8$. We show in Figure 8 the mean winning bid in the simulation (solid curve) together with the real result of the auction (dashed curve).

We can observe a globally good agreement between the output of the simulation and the real data in the sense that the simulated prices follow the trends of the real ones. This is true for the 24 rounds between 2017 and 2022 with two exceptions around February 2020 and February 2021. They might be explained first by the covid epidemic and the general economic lock-down that took place from February 2020 which generated unusual conditions. Around February 2022, we can first notice a one round larg between the real and simulated prices and also that the prices fall is much more pronounced in the simulation. At the same time, just before Feb. 2020 and Feb. 2022, we can notice the brutal raise of the competition level, which resulted in the prices falling in the simulation but not in reality. A possible explanation would be that some bidders may have insights on the competition level of the next round, either anticipating the volume the government will auction or that some bidders will not participate.

As for the German wind on-shore auction, the data for the solar PV auction are available on the website the German government Federal Ministry for Economic Affairs and Climate Action. However, contrary to the wind on-shore auction, the total submitted volume is not available, thus preventing the computation of the competition level. We obtained it from Figure 3 in Szabó et al. (2020) which explaines why we limit our study to the years 2015-2019. The best fit was obtained supposing that participants have all a cost equal to 4 with a μ initially uniformly distributed between 9 and the ceiling price of the first round, and the learnning rate is $\gamma = 0.15$. Real and simulated prices are shown in Figure 9, together with the real ceiling price and competition level. Again we can appreciate that the simulate prices globally follow the tendency of the real prices, though the variations in the real prices are more pronouned that in the simulation. They both went down from the start in April 2015 to Feb. 2018 when the competition level was clearly greater than 2. From June 2018 to Feb. 2019, the real and

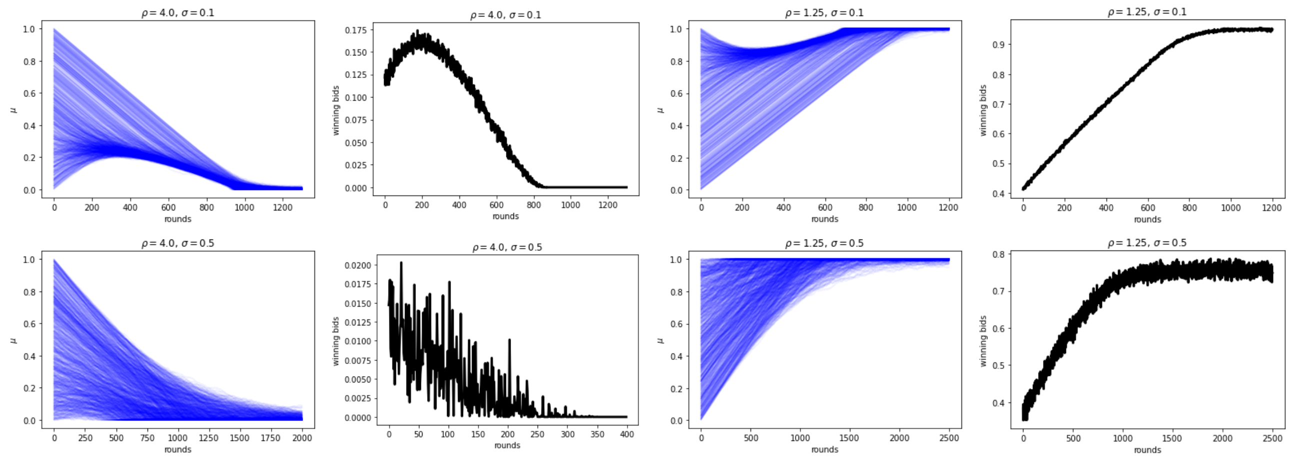

Figure 7: Evolution from round to round of the μ values of bidders (blue curves - columns 1 and 3), and of the mean winning bid (black curve - columns 2 and 4) for two constant competition level values $\rho = 4$ (columns 1 and 2) and $\rho = 1.25$ (columns 3 and 4), and for two different values of σ: $\sigma = 0.1$ (top) and $\sigma = 0.5$ (bottom). Parameter μ is initially uniformly distributed among the bidders.

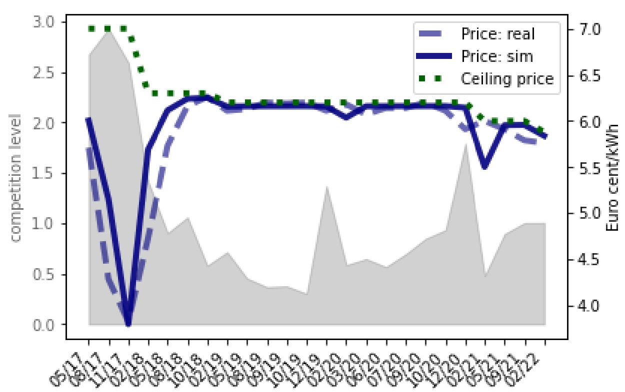

Figure 8: Evolution from round to round of the competition level ρ defined in (1), the ceiling price (red dotted line), the real mean winning bid (dashed light blue line) for the wind on-shore auctions in Germany (based on data available at Bundesnetzagentur), and the mean winning bid price in our model (solid blue line) for the parameters given in the text.

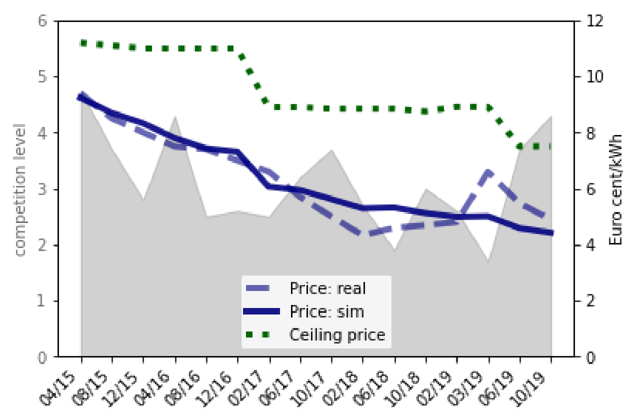

Figure 9: Evolution from round to round of the competition level ρ defined in (1), the ceiling price (red dotted line), the real mean winning bid (dashed light blue line) for the solar PV auctions in Germany (based on data available at Bundesnetzagentur), and the mean winning bid price in our model (solid blue line) for the parameters given in the text.

simulated prices exhibit opposite tendencies (the real price rose, the simulated fell) though the relative price variation over this period is low. This might also be caused by some bidders having insights about the future level of competition. Then from Feb. 2018 on, both the real and simulated prices show the same trend.

5.2. Argentine solar PV auction.

In order to increase the share of RE in the energy mix, the Argentine government lauched in 2017 an auction scheme called RenovAr which consisted of 3 rounds (called round 1, 1.2 and 2) of technology-specific sealed-bid auction. The RenovAr scheme is generally considered a success in view of the low prices that were obtained, a consequence of the high level of participation. In fact the overbidded round 1 led to the creation of an initially unplanned round 1.5. The implementation of a guarantee mechanism efficiently helped to mitigate the perceived risks associated with the economic and political instabiity of the country. An extensive account of the Argentine energy market and a description of this scheme can be found in Menzies, Marquardt and Spieler (2019).

We will concentrate on the solar PV auction for which we had access to detailed data (Prioletta (2022)). We report in Table 1 the ceiling price, auctionned and bidded volume of round 1, 1.5 and 2. Figure 10 shows the competition level (grey), ceiling price (green dotted line) and mean price (dashed light blue) that were obtained.

A close examination of the bidders population reveals (Prioletta (2022)) that the participants of Renovar Solar PV can be split into four groups according to the range of volume bidded (see Table 2). We report in Tables 3 and 4 the mean and variance of the LCOE and of the bids of the 1st round.

Round 1.5 was not initially planned and was created in response to the largely overbidded round 1 where the level of competition was equal to 600 % (i.e. the bidded volume was approximately 6 times higher that the auctionned volume). However only the loosers of round 1 were allowed to participate to round 1.5. We cannot therefore use our model to simulate the three rounds at once. Instead we run it round by round explaining the adjustment we made at each round. Notice that we retain the core idea of our model, namely all the bidders are myopic and update their relative mark-up according to their result in the round.

We iniciate the simulation of the ABM model choosing independently at random the cost and the volume bidded of each of the 50 participants of round 1 depending on his group using the following distribution: bidders' costs are drawn from a normal distribution with mean and variance given in Tables 3, and the volume bidded is chosen uniformly between the minimum and maximum value given in Table 2). Bidders' μ parameter are chosen independently at random from a normal distribution with mean and variance given in Tables 4. Steps 2 (bidding step) and 3 (winner determination) of our model are then executed as explained in section 2. We chose $\sigma = 0$ since the generation of the μ's is already noisy. All bidders then adjust their relative mark-up (see step 4). In all the simlation we took a learning rate $\gamma = 0.26$ since it resulted in a the better fit. This ends the simulation of round 1. Following the rule applied in Renover, the ceiling price of round 1.5 was taken as the mean winning bid of round 1. As explained before, only the loosers of round 1 could participate to round 1.5. Their cost was drawn at random from a normal distribution with mean and variance as given in Table 3, and their μ was then computed as in formula (2) using the relative mark-up updated at the end of round 1. We then proceeded with Step 2, 3 and the updating of the relative mark-up. The ceiling price of round 2 was computed as the mean between the mean winning bid of round 1 and 1.5 as in Renover. The cost of the 50 participants was drawn at random from a normal distribution with mean and variance given in Tables 3, and participants μ parameter was computed using the updated relative mark-up. New participants were added: 10 in group 1, 3 in group 2, 1 in group 3, and 13 in group 4, whose cost were drawn from a normal as before, and whose μ were chosen unformly between their cost and the ceiling price of round 2. We then proceeded with step 2 and 3 of the simulation.

After averaging over 100 runs, we obtained the simulated ceiling price (dashed green line) and mean prices (solid blue line) shown in Figure 10. We can appreciate the very good agreement between the simulation and the reality both for the ceiling price and the prices.

Table 1
Ceiling price, auctionned and bidded volume in RenovAr Solar PV.

Round	Ceiling price	Auctionned Vol.	Bidded vol.
1	90	400	2304.21
1.5	59.75	517	876.98
2	57.04	817	3989

Table 2
Minimum and maximum bidded volume by groups in RenovAr Solar PV.

Bidded volume	Group 1	Group 2	Group 3	Group 4
min	2	21	80	100
max	20	50	80	100

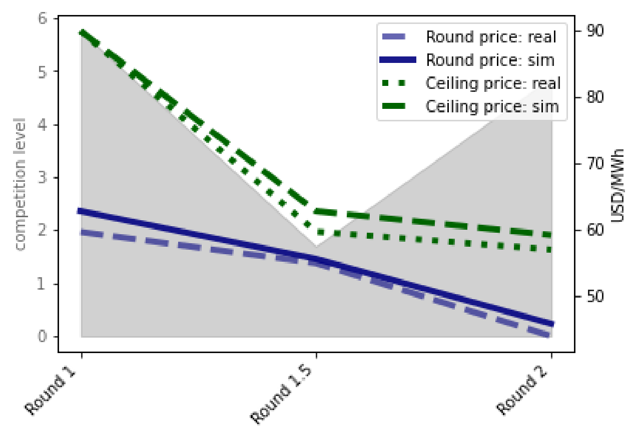

Figure 10: Evolution from round to round of the competition level ρ defined in (1), the ceiling price (real and simulated), and the prices (real and simulated) for the solar PV auctions in Argentina.

6. Exploring policies to ensure a high enough level of competition.

We saw in the previous section that our model can reproduce the trends of the prices in real auctions thus confirming that a a ratio of at least 2 to 1 between the total volume offered by bidders and the auctionned volume is critical to obtain low prices. Several regulations can be implemented toward that objective whose impact can be assessed with our model.

The government can first act on the level of competition *indirectly* by promoting participation with the help, for instance, of fiscal incentive, mechanism to mitigate perceived risks as in RenovAr, or reduced finantial guarantees like the advantages small energy producers benefited from until the end of 2017 in the German scheme.

The government can also act *directly* on the level of competition decreasing the auctionned volume. For instance on 22 May 2019, Ukraine adopted a law to introduce auctions as the main instrument to deploy renewable energy.

Table 3
LCOE (mean and variance) for each round of Renovar Solar PV for each group of bidders.

Round	Group 1		Group 2		Group 3		Group 4	
	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
1	61.1	35.2	62.3	63	52.0	9.8	51.9	59.8
1.5	54.8	3.3	53.9	12.9	49.7	1.2	56.8	0.9
2	47.1	20.7	48.4	22.8	45.4	54.1	45.7	22.4

Table 4
Mean and variance of the bids received in the round 1 of Renovar Solar PV for each group of bidders.

	Group 1		Group 2		Group 3		Group 4	
	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
Bid	80.3	75.3	83.4	81.9	74.6	155.5	71.8	138.3

According to Anatolitis and Grundlach (2020) Ukrainian authority thought of implementing a scheme in which the total awarded capacity may not exceed 80% of the total volume offered by the bidders. This is quite an unusual rule which is explicitly aimed at increasing competition among bidders. In our framework, if in a given round a volume V was initially put in auction, applying this rule means that the effectively auctionned volume is $V = Nv \times 80\%$. The level of competition ρ defined in 1-3 is then $\rho = \frac{Nv}{Nv \times 80\%} = 1.25$. The Ukrainian rule thus aims at ensuring a competition level of at least $\rho_0 = 1.25$. In particular it is best to apply it when $\rho < \rho_0$. Since $1.25 < 2$, we do not expect low prices, which is confirmed by the simulation shown in Figure 11 where the prices (solid orange line) are close to the real prices.

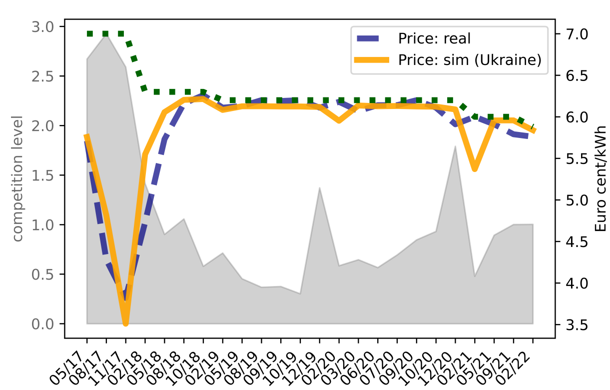

Figure 11: Evolution of the prices in the German wind on-shore auction when acting on the level of competition using the "80% Ukrainian rule" (dashed orange) The ceiling price is the same as in the real German auction.

Elaborating on this idea, and taking advantage of our finding about the critica impact of having a competition level greater or lower than 2, the government could choose to decrease the auctionned volume so as to reach a desired level of competition $\rho_0 > 2$ when the competion level is below 2. This is the Ukrainian 80% rule with 1.25 replaced by $\rho_0 > 2$. To achieve a competition level ρ_0 when the competition level ρ is less than 2, we just have to auction a new volume $V' = \frac{\rho}{\rho_0} V$ since in that case the new competition level is $\frac{Nv}{V'} = \rho_0 \times (Nv/V)/\rho = \rho_0$ as desired. Taking e.g. $\rho_0 = 2.1$ we obtain the prices (solid dark red line) shown in Figure 12. We can see that ensuring a competition level of at least $\rho_0 = 2.1$ by decreasing the auctionned volume leads as expected to lower and more stable prices.

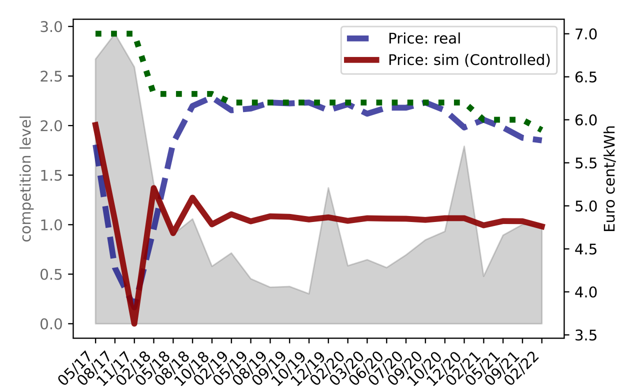

Figure 12: Evolution of the prices in the German wind on-shore auction when acting on the level of competition using the "Ukraine rule" (dashed orange) The ceiling price is the same as in the real German auction.

Notice that increasing the level of competition from ρ to ρ_0 is achieved reducing the offered volume V down to V' in proportion $V'/V = \rho/\rho_0$. Such a reduction of adjudicated volume may be significant if the competition level ρ is initially low, thus perturbing the overall RE deployment planning. In the case of the German wind on-shore auction, we show in Figure 13 the accumulated auctionned volume (blue), the real accumulated adjudicated volume (orange), and the accumulated adjudicated volume resulting from the optimization rule to have $\rho = \rho_0$ (green) which is significanlty less than the auctionned volume. This problem may however be compensated increasing the frequency of auctions.

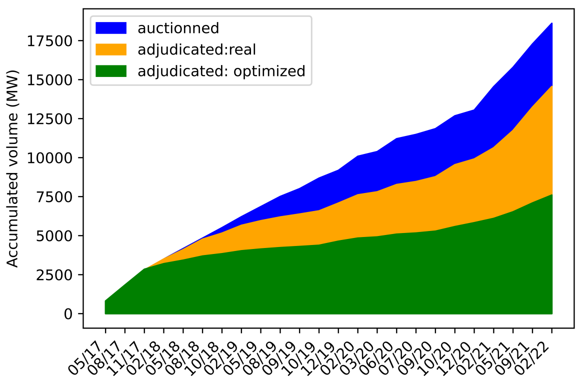

Figure 13: Evolution of the accumulated volume in the German wind on-shore auction: auctionned volume (blue), real adjudicated volume (orange), adjudicated volume when acting on the level of competition to enforce $\rho = \rho_0 = 2.1$.

In this simulation we kept the ceiling price as in the real wind on-shore German auction. In reality the ceiling price was however computed dynamically from round to round taking into account the results in the past rounds using the following rule (see Sach et al. (2019)[§3.2.1]): the celing price at round t is the mean between the maximum winning bid price of the previous threee rounds then increased by 8%. Using this rule in our model yields however an increase of the prices. Indeed due to the low level of competition, most bidders wins and thus increase their bid from round to round without upper limit. We can prevent this simply taking the minimum between the ceiling price of round t-1 and the new ceiing price computed with the above formula. Together with the control of the competition level with $\rho_0 = 2.1$, we obtain the figure 14, very similar to figure 12 obtained previously.

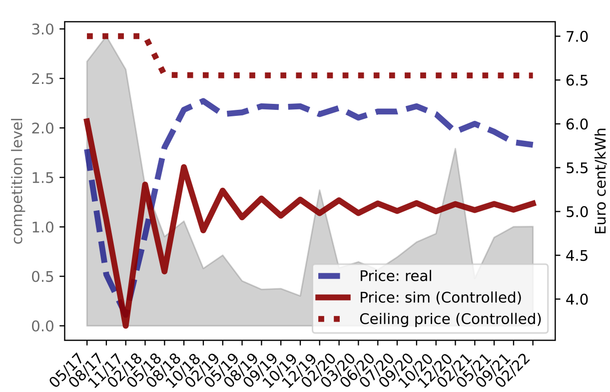

Figure 14: Evolution of the prices in the German wind on-shore auction when acting on the level of competition to ensure $\rho \geq 2.1$ ("solid dark red). The ceiling price is adjusted automatically from roun to round following the German rule.

Others rules for the ceiling price can be considered. For instance in RenovAr, the ceiing price of a round was the mean price of the previous round. We can then mix the Argentine and German rule e.g. computing the ceiing price of a round t as the mean price of the previous round increased by 8%, say, keeping the minimum value with the ceiling price of the previous round to ensure a decreasing ceiling price. In that case we obtain the result shown in Figure 15 according to which this new mixed rule for computing the ceiling price yields prices much lower than the German rule.

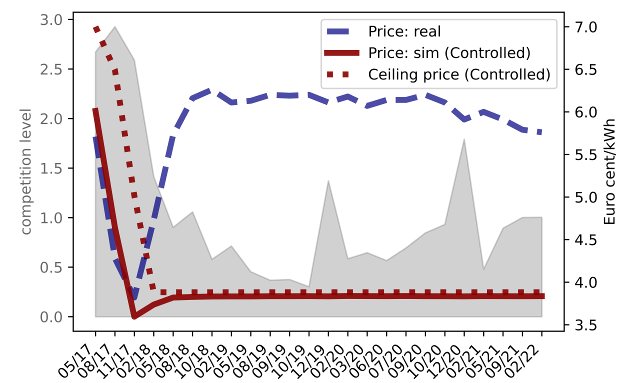

Figure 15: Evolution of the prices in the German wind on-shore auction when acting on the level of competition to ensure $\rho \geq 2.1$ (solid dark red). The ceiling price is adjusted automatically from round to round as described in the text (mix of the German and Argentine rule).

With these examples we do not pretend performing a complete study of the impact on prices of dynamic ceiling prices but only draw attention on the interest of controlling the competition level by acting on the auctionned volume together with a dynamic ceiling prices to achieve low prices. We also want to emphasize that our model seems to be an appropriate framework to assess the impact of these design parameters on the prices.

7. Conclusion and Policy Implications

We proposed in this paper a simple model of a procurement auction taking place in several rounds where a large number of bidders offer at each round a fixed volume v of energy. At each round only a given proportion $1/\rho$ of participants win. We can then think of ρ as the competition level of the round. The participants' bidding behaviour, i.e., the price they offer for a volume v, is characterized by two parameters, namely the mean bid value μ and its deviation. From round to round, participants myopically adjust their mean bid value μ acting on their relativa mark-up (the profit they could realize relatively to their cost and the ceiling price): they raise it or lower it by a constant increment γ if they are among the winners or losers, respectively, of the round.

Agent-based simulation shows that whatever the initial distribution of bidders' μ parameter, they eventually coordinate in the sense they end up using the same μ value (up to negligible random fuctuations) which is moreover either close to the ceiling price or close to the cost depending on the level of competition. The dynamics of our model is thus in fact essentially governed by one parameter, namely the competition level ρ. Indeed the learning parameter γ only affects the time scale and σ only seems to delay the coordination. This is thus a suitable framework to study theoretically the impact of the competition level.

It is well known in the literature and among practitioners that the competition level among bidders is an important factor in the success or failure of an auction with respect to the final price (see Bulow and Klemperer (1994); Wigand et al. (2016); Kreiss (2016)). Examining together prices and competition level in the German wind on-shore and solar PV auctions show indeed that a competition level of 2 seems to be a critical threshold for th evolution of prices. Our model confirms this empirical finding and quantifies precisely the impact of the competition level ρ on the evolution of prices. Indeed agent-based simulations show the presence of a phase transition at $\rho = 2$: prices increase to the ceiling price when $\rho < 2$, whereas they drop down to the true costs when $\rho > 2$.

Moreover our model can reproduce reasonably well the trends in the evolution of prices observed in three real auctions, namely Germany wind on-shore and solar PV, and Argentine solar PV auctions. To our knowledge, this is the first model of procurement auctions able to reproduce real data.

Policy makers thus must take actions to ensure a high enough level of competition, which is according to our findings a ratio of at least 2 to 1 between the total volume offered by bidders and the total volume put to auction by the auctioneer. Several regulations can be implemented toward that objective whose impact can be assessed with our model. For instance simulations in the setting of the German wind on-shore auction suggest that controlling the auctionned volume to ensure a competition level of at least $\rho_0 > 2$ together with a dynamic ceiling price (e.g taking the mean of the price of the previous round increased by some percentage) lead to low and stable prices. Computing the ceiling price dynamically has already been used and do not seem to present any particular difficulty to implement. However modifying the auctionned volume in view of the bidded volume is quite uncommon and so must be handled carefully. Notice also this kind of rule to adjust the auctionned volume presents a clear drawback: the adjudicated volume at each round can be significantly less than the initial planned auctionnd volume, which can be a serious problem when the country has urgent energetic needs. One possible solution would be to increase the frequence of rounds.

As a final comment, we believe that the model presented in this paper is a suitable framework to further study theoretically the impact of auction design parameters others than the competition level, thus complementing existing data-oriented literature. We will consider different kinds of participants structured according to their economic capacity in the spirit of Anatolitis and Welisch (2017). Also, we considered in this paper myopic bidders only. We also plan to incorporate to our model bidders behaving rationally (or partially rationally) thus following the classic economic literature on bounded rationality. The importance of the payment mechanism and bidder's evaluation of the project risks are also important aspects that will be addressed in future works.

CRediT authorship contribution statement

Nicolas Saintier: Conceptualization of this study, Methodology, Software, Writing and Editing. **Javier Marenco:** Conceptualization of this study, Methodology, Software, Writing and Editing. **Martin Kind:** Conceptualization of this study, Methodology, Software, Writing and Editing. **Juan Pablo Pinasco:** Conceptualization of this study, Methodology, Software, Writing and Editing.

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